Minimizing Wait Latency
in Periodic P2P Hypercube Gossiping

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Gossiping

- Elemental Dissemination Problem

- Aka. *Total Exchange*

- Communicate every Node's Information to all other nodes

- Periodic: Live streaming between Peers
Applications

- Audio Communication for MMVEs
- Decentralized Data Fusion for Object Tracking
- Real-time Business Process Intelligence
- Agent-based Management for Smart-Grids
- Shared Haptic Virtual Environments
- ...
Hypercube Gossiping

- $H_m$: Binary Hypercube of dimension $m$
  - $2^m$ nodes
  - Diameter $m$
  - Degree $m$
  - Gossip complexity $m$
Issue: Latency

- Real-time applications require current data

- Traversal time depends on
  - Underlay Network Delay
  - Overlay Hop Distance
  - Wait Latency
Underlay Network Delay

![Network Delay Distribution]

- **Median**: 45 ms
- **Mean**: 61 ms

**Network Delay [ms]**

0 40 80 120 160 200 240 280 320 360 400

**Density**

0.000 0.005 0.010 0.015 0.02
# Overlay Distance

\[
c(L, h) = \binom{L}{h}
\]

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<thead>
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<th>(c(L, h))</th>
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Wait Latency
Timed Sequence Graphs

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Wait Latency Measures

- **Path Latency** \( \Delta_{i,j} \)
- **Maximum Latency** \( \hat{\Delta} = \max (\Delta_{i,j}) \)
- **Overall Latency** \( \Delta_{tot} = \sum_{i=0}^{N-1} \sum_{j=0, j\neq i}^{N-1} \Delta_{i,j} \)
- **Mean Latency** \( \overline{\Delta} = \frac{\Delta_{tot}}{N \cdot (N - 1)} \)
- **Expected Mean Latency** \( E[\Delta] \)
Timing Modes

Random Mode:

\[ \hat{\Delta}_{\text{Random}} = (L + 1)T \]
\[ E[\Delta_{\text{Random}}] = \frac{1}{2} \left( 2^{L-1}L + 2^L - 1 \right) T \]

Sync Mode:

\[ \hat{\Delta}_{\text{Sync}} = L \cdot T \]
\[ E[\Delta_{\text{Sync}}] = \frac{2^{L-2}L^2}{2^L - 1} T \]

Spliced Mode:

\[ \hat{\Delta}_{\text{Spliced}} = (L - 1 + \frac{1}{k})T \]
\[ E[\Delta_{\text{Spliced}}] = \frac{1}{2} \left( \frac{1}{2}L - 1 + \frac{1}{k} \right) \frac{2^L - 1}{2^L - 1} T \]

Chained Mode:

\[ \hat{\Delta}_{\text{Chained}} = \left( \frac{L - 1}{k} + \frac{L - 1}{2} \right) \left( 1 - \frac{2}{k} \right) T \]
\[ E[\Delta_{\text{Chained}}] = \frac{1}{2} \left( \frac{1}{2}L \cdot 2^L - 1 \right) \frac{1}{k} + g(L) \left( 1 - \frac{1}{k} \right) T \]

Chained Mode

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Complementary Delays

\[ \delta_a(x) \]

\[ \delta_b(x) \]

\[ \delta_a(x) + \delta_b(x) \]

\[ ((b - a) \mod T) + T \]

\[ (b - a) \mod T \]
A Slice of Modulo Space

\[ 0 = T \mod T \]

\[ b - a \]

\[ [a, b]_T \]

\[ b \]

\[ a \]

\[ z \in [a, b]_T \]

\[ (b - z) \mod T \leq (b - a) \mod T \]

\[ [a, b]_T := \begin{cases} 
[a, b] & \text{for } a < b \\
[0, b] \cup [a, T] & \text{for } a > b \\
\{\} & \text{for } a = b 
\end{cases} \]
Primary and Secondary Offsets

Offsets to reference node:

\[ x_i := (\varphi_{\alpha+2(i+1),0} - \varphi_{\alpha,0}) \mod T, \; i \in [0..N^* - 1] \]

Offsets between any two nodes:

\[ z_k \in \left\{ x_i | i \in [0..N^*] \right\} \cup \left\{ x_j - x_i | i, j \in [0..N^*] \land j > i \right\} \]
Primary Offsets

Case $b \geq a$

\[-z \leq -a\]
\[z \leq b\]

Case $b < a$

\[-z - T \cdot c \leq -a\]
\[z + T \cdot c \leq b + T\]

$c \in \{0; 1\}$
Secondary Offsets

Case $b \geq a$

Case $b < a$

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The Shape of the Perfect Solution

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Case $z$ is primary offset:

Case $b > a$:

\[-z \quad -T \cdot p \quad \leq \quad -a\]

\[z \quad -T \cdot p \quad \leq \quad b\]

Case $b < a$:

\[-z \quad -T \cdot c \quad -T \cdot p \quad \leq \quad -a\]

\[z \quad +T \cdot c \quad -T \cdot p \quad \leq \quad b + T\]

\[c \in \{0, 1\}\]

Case $z$ is secondary offset:

Case $b > a$:

\[-z \quad -T \cdot c \quad -T \cdot p \quad \leq \quad -a\]

\[z \quad +T \cdot c \quad -T \cdot p \quad \leq \quad b\]

\[c \in \{0, 1\}\]

Case $b < a$:

\[-z \quad -T \cdot c \quad -T \cdot p \quad \leq \quad -a\]

\[z \quad +T \cdot c \quad -T \cdot p \quad \leq \quad b + T\]

\[c \in \{0, 1, 2\}\]

\[w = 2^{L-(r-l)-2}\]

\[\text{obj} = T \sum w_e p_e\]
lp_solve

- Open Source Solver
- Michael Berlelaar Eindhoven University of Technology
- Enhanced and updated by various Individuals
- Linear and Mixed Integer Optimizer
  - Uses branch and bound algorithm
  - Presolve algorithms pre-processes the problem to reduce problem size and solve time.
  - Handles binary, integer and semi-continuous variables, and special ordered sets.
  - Customizable node and variable selection strategies.

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Results

Reduction of Wait Latency in Crossing Mode for Different Network Sizes, 5 Minutes Time Limit for lp_solve

Number of Nodes

256 128 64 32 16 8

Percent Reduction with regard to Chained Mode
Summary & Future Work

- Hypercube gossiping facilitates efficient aggregation for a broad range of applications
- Real-time applications require highly current data
- High portion of latency due to wait delay, i.e. data sojourn times at intermediate nodes
- Depends on timing behavior of nodes
- Modeling of Wait-Latency as a MILP-Optimization Problem
- Significant reduction of wait latency
- Performance of the employed solver currently limits perfect solution to 64 nodes

Thank you!