Minimizing Wait Latency in Periodic P2P Hypercube Gossiping

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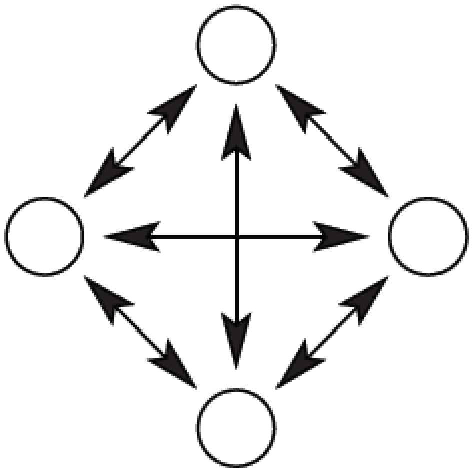
Gossiping

- Elemental Dissemination Problem
- Aka. *Total Exchange*
- Communicate every Node's Information to all other nodes

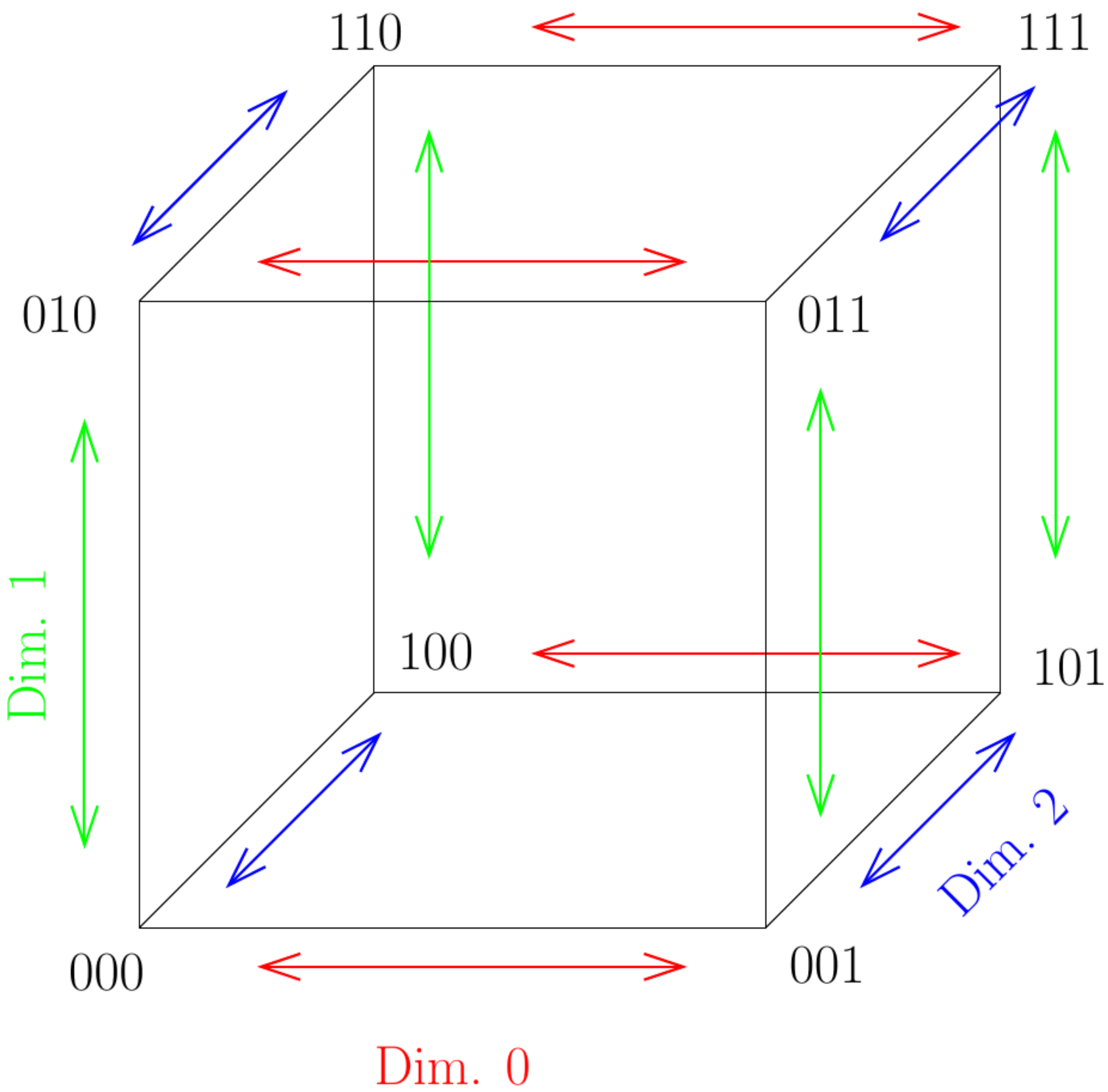


- Periodic: Live streaming between Peers

Applications

- Audio Communication for MMVEs
- Decentralized Data Fusion for Object Tracking
- Real-time Business Process Intelligence
- Agent-based Management for Smart-Grids
- Shared Haptic Virtual Environments
- ...

Hypercube Gossiping

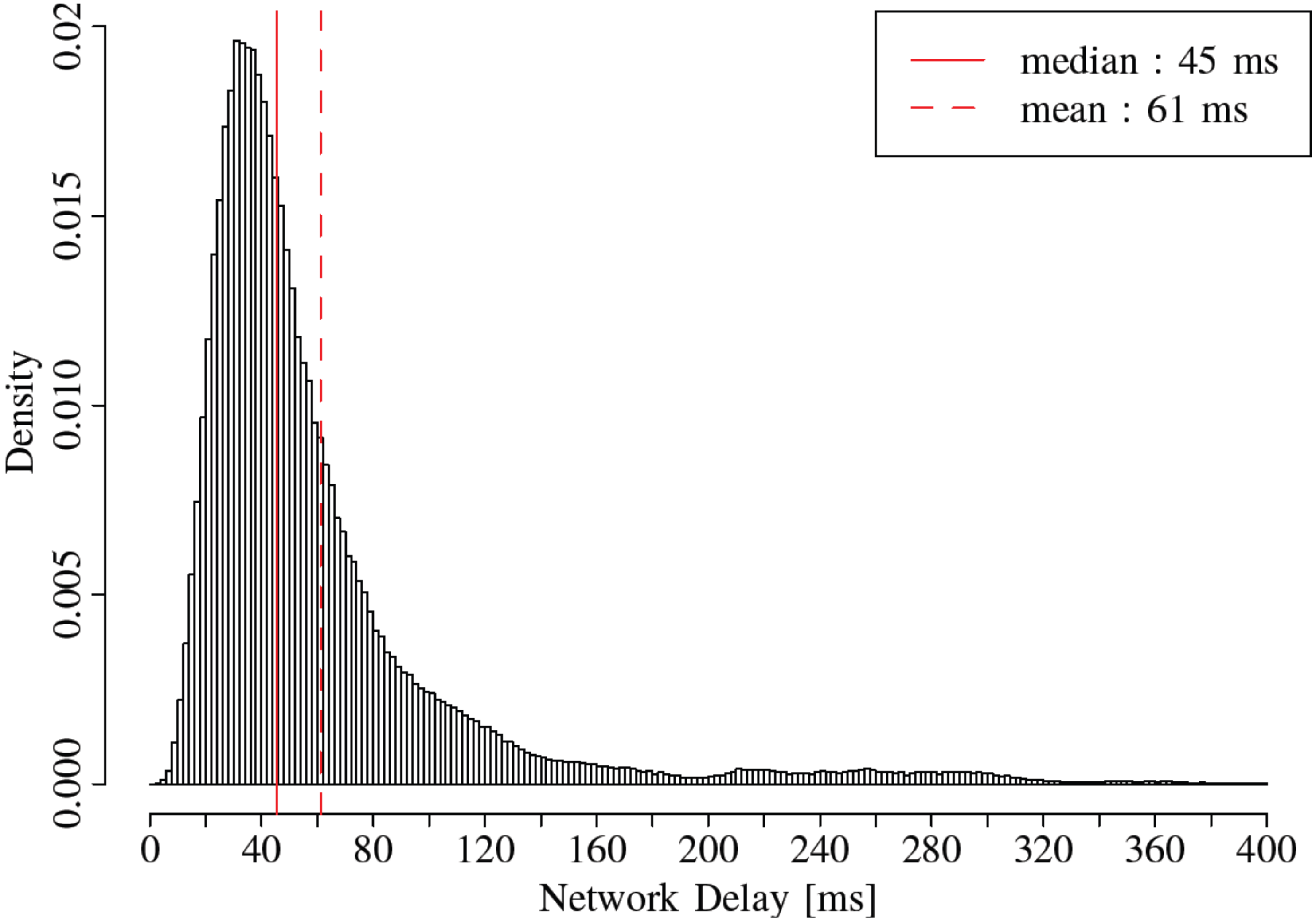


- H_m: Binary Hypercube of dimension m
  - 2^m nodes
  - Diameter m
  - Degree m
  - Gossip complexity m

Issue: Latency

- Real-time applications require current data
- Traversal time depends on
  - Underlay Network Delay
  - Overlay Hop Distance
  - Wait Latency

Underlay Network Delay



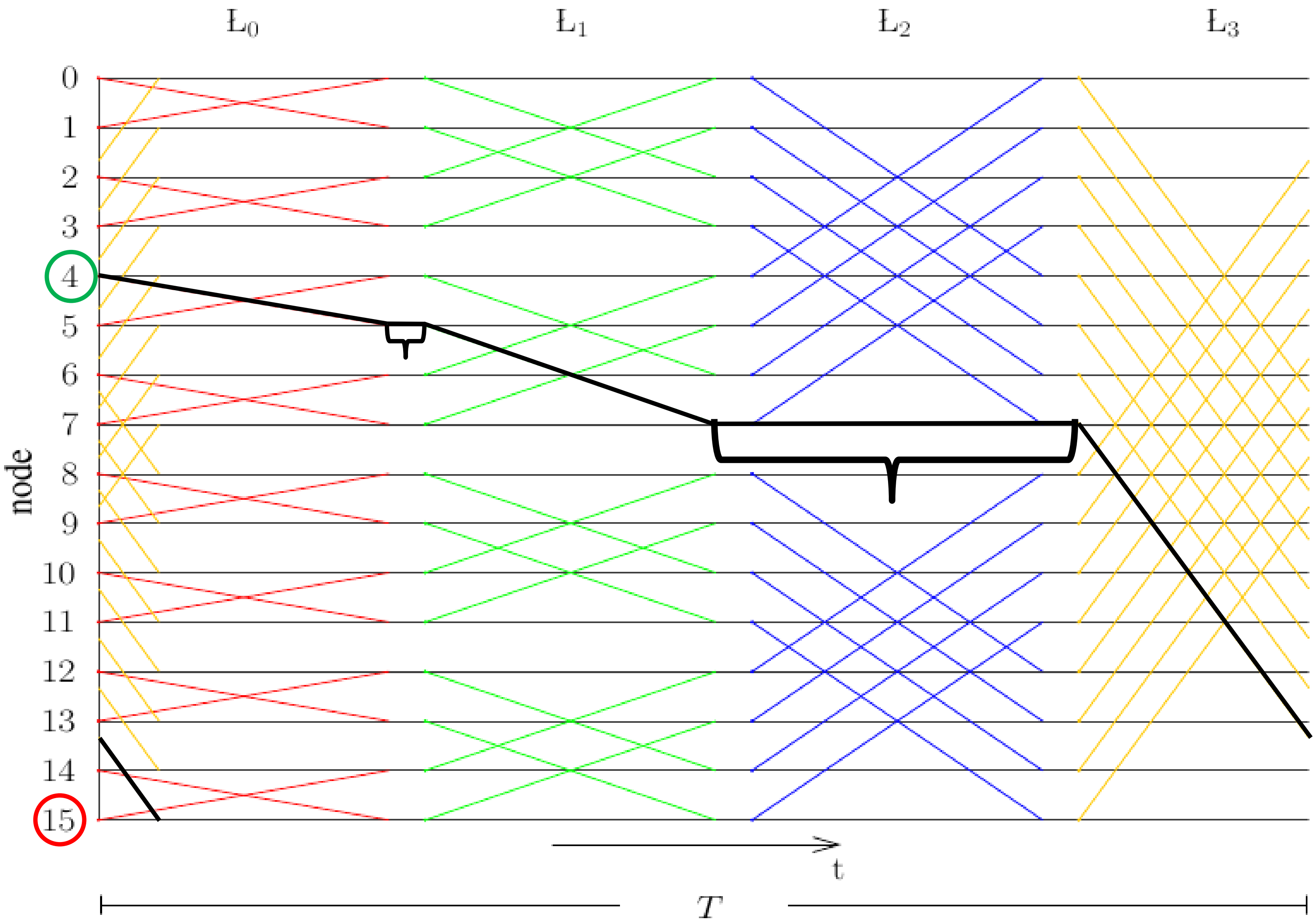
Overlay Distance

$$c(L,h) = \binom{L}{h}$$

$c(L,h)$	h							
L	1	2	3	4	5	6	7	8
1	1	0	0	0	0	0	0	0
2	2	1	0	0	0	0	0	0
3	3	3	1	0	0	0	0	0
4	4	6	4	1	0	0	0	0
5	5	10	10	5	1	0	0	0
6	6	15	20	15	6	1	0	0
7	7	21	35	35	21	7	1	0
8	8	28	56	70	56	28	8	1

Wait Latency

Timed Sequence Graphs

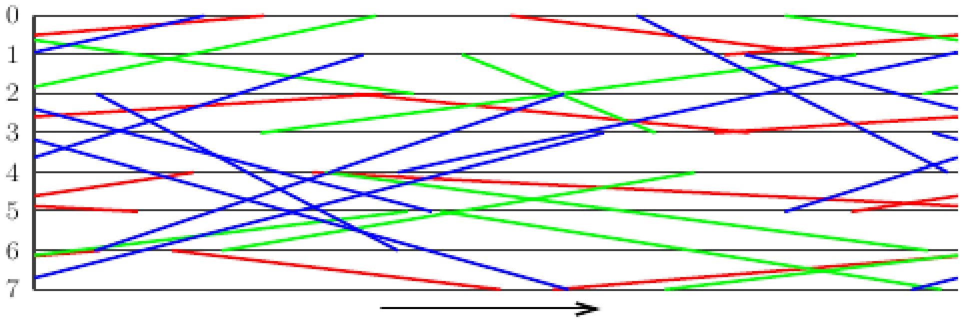


Wait Latency Measures

- Path Latency $\Delta_{i,j}$
- Maximum Latency $\hat{\Delta} = \max(\Delta_{i,j})$
- Overall Latency $\Delta_{tot} = \sum_{i=0}^{N-1} \sum_{j=0, j \neq i}^{N-1} \Delta_{i,j}$
- Mean Latency $\overline{\Delta} = \frac{\Delta_{tot}}{\mathrm{N} \cdot (\mathrm{N}-1)}$
- Expected Mean Latency $E\left[\overline{\Delta}\right]$

Timing Modes

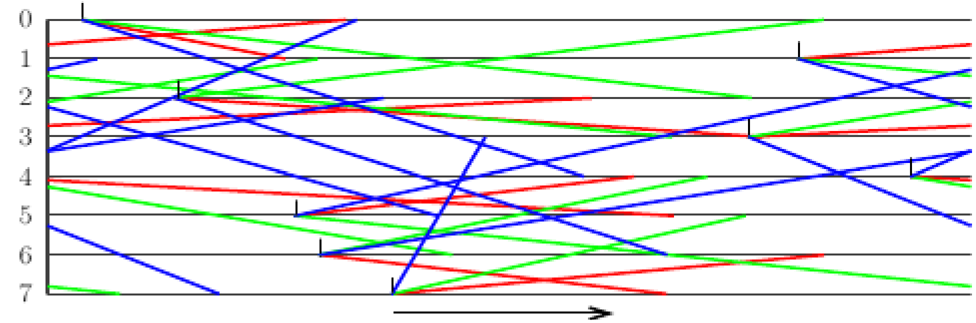
Random Mode:



$$\hat{\Delta}_{Random} = (L+1)T$$

$$E\left[\Delta_{Random}\right] = \frac{1}{2}\frac{2^{L-1}L + 2^{L} - 1}{2^{L} - 1}T$$

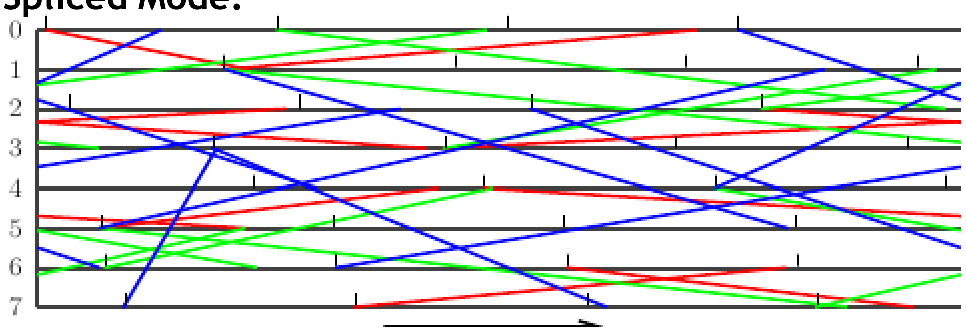
Sync Mode:



$$\hat{\Delta}_{Sync} = L \cdot T$$

$$E\left[\Delta_{Sync}\right] = \frac{2^{L-2}L}{2^{L} - 1}T$$

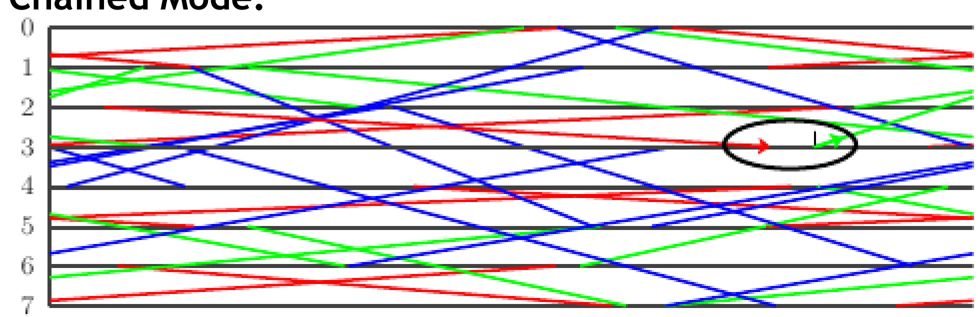
Spliced Mode:



$$\hat{\Delta}_{Spliced} = (L - 1 + \frac{1}{k})T$$

$$E\left[\Delta_{Spliced}\right] = \frac{1}{2}\frac{\left(\frac{1}{2}L - 1 + \frac{1}{k}\right)2^{L} - 1}{2^{L} - 1}T$$

Chained Mode:

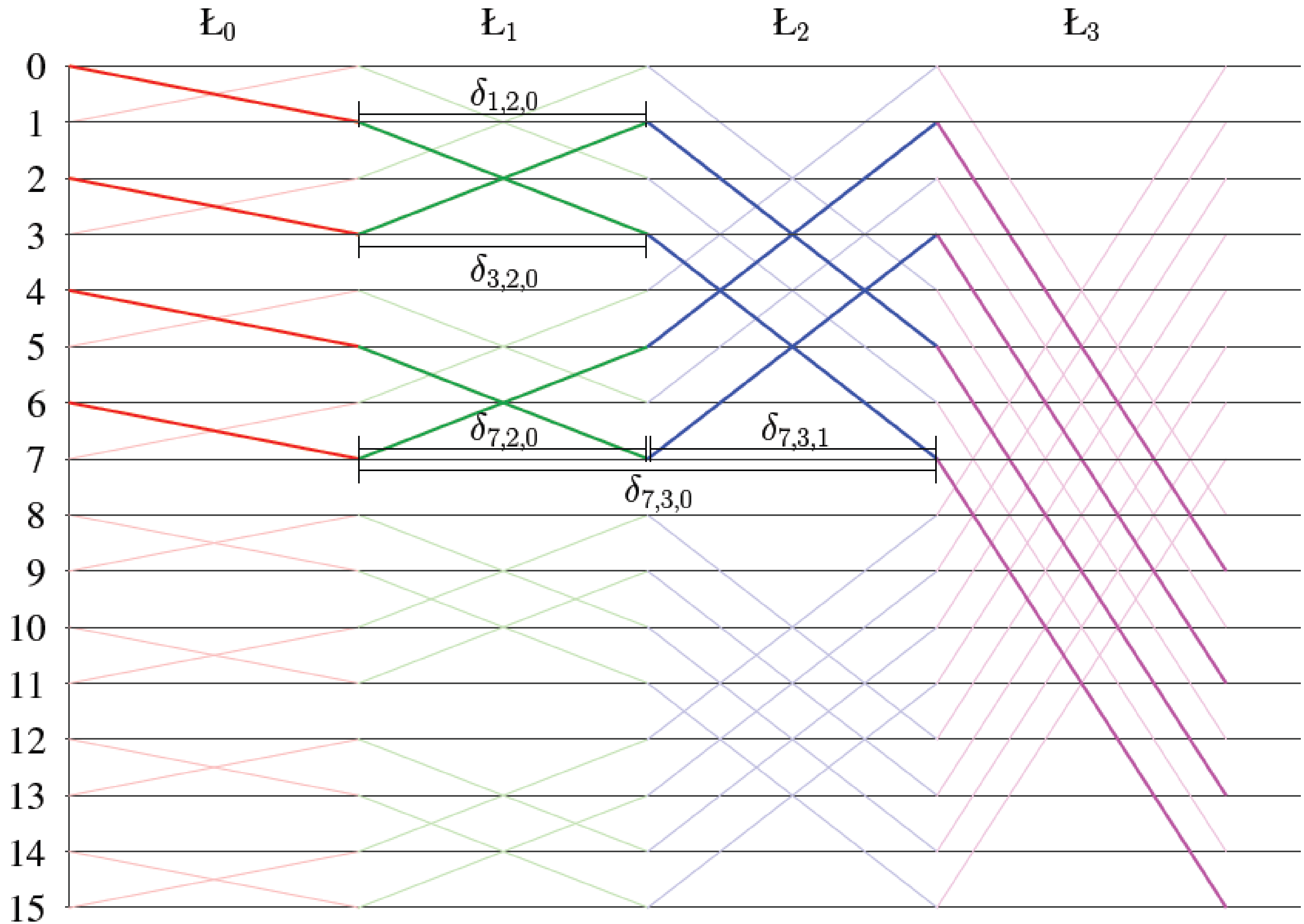


$$\hat{\Delta}_{Chained} = \left(L \cdot \frac{1}{k} + \left\lfloor \frac{L-1}{2} \right\rfloor \left(1 - \frac{2}{k}\right)\right)T$$

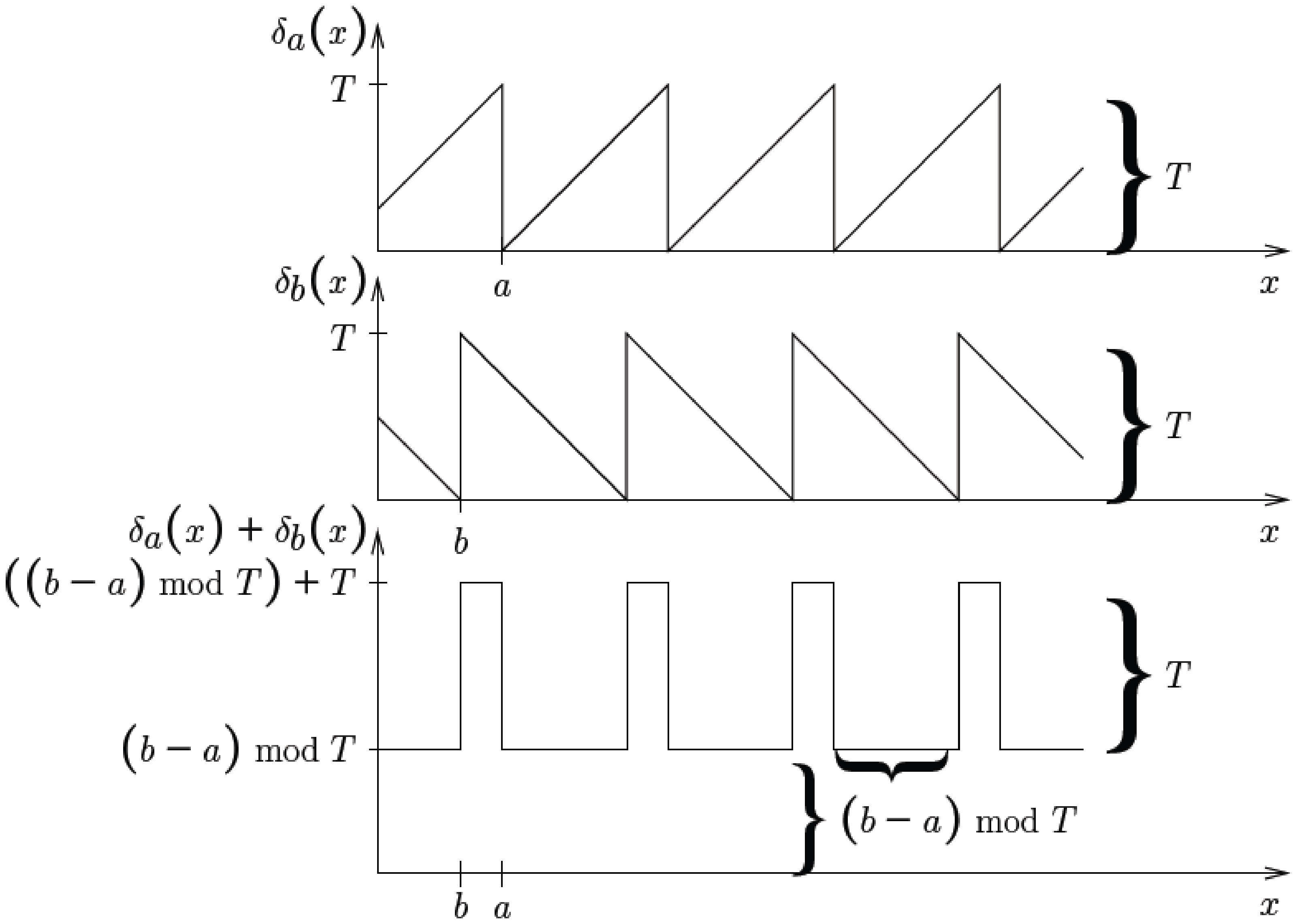
$$E\left[\Delta_{Chained}\right] = \frac{1}{2}\frac{\left(\frac{1}{2}L \cdot 2^{L} - 1\right)\frac{1}{k} + g(L)\left(1 - \frac{1}{k}\right)}{2^{L} - 1}T$$

[1] Characterizing Latency in Periodic P2P Hypercube Gossiping, 4th Intl. Conf. on Communications Systems and Networks (COMSNETS), January 2012.

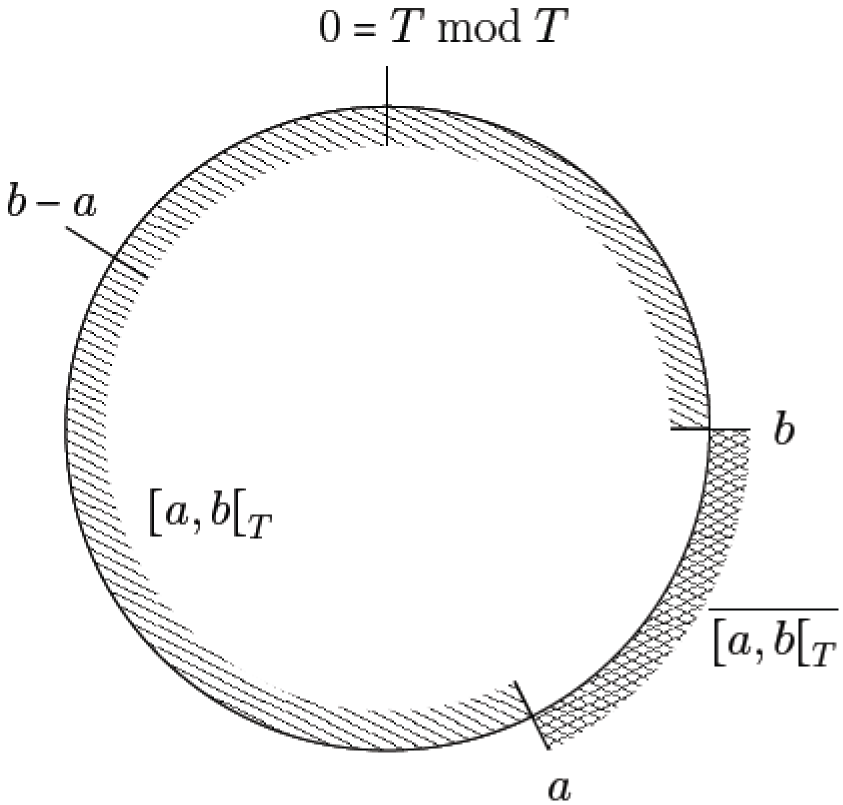
Chained Mode



Complementary Delays



A Slice of Modulo Space



$$z \in [a, b[_T \iff (b - z) \bmod T \le (b - a) \bmod T$$

$$[a, b[_T := \begin{cases} [a, b[& \text{for } a < b \\ [0, b[\cup [a, T[& \text{for } a > b \\ \{\} & \text{for } a = b \end{cases} \quad \text{with } a, b \in [0, T[$$

Primary and Secondary Offsets

Offsets to reference node:

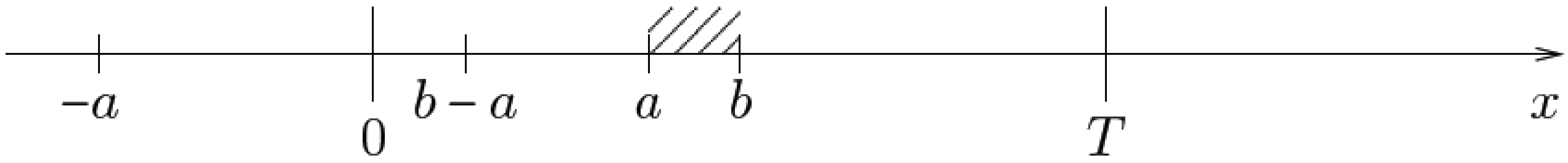
$$x_i := \left(\varphi_{\alpha+2(i+1),0} - \varphi_{\alpha,0}\right) \bmod T,\ i \in [0..N^* - 1[$$

Offsets between any two nodes:

$$z_k \in \left\{x_i | i \in [0..N^*[\right\} \ \cup \ \left\{x_j - x_i | i, j \in [0..N^*[\wedge j > i\right\}$$

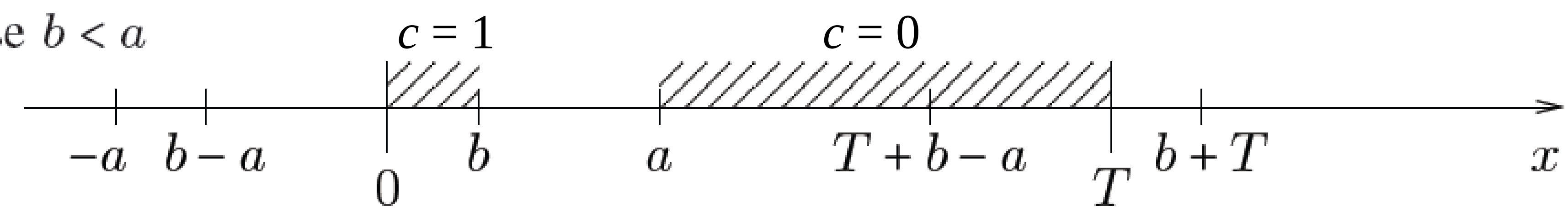
Primary Offsets

Case $b \geq a$



$$\begin{aligned} -z &\leq -a \\ z &\leq b \end{aligned}$$

Case $b < a$

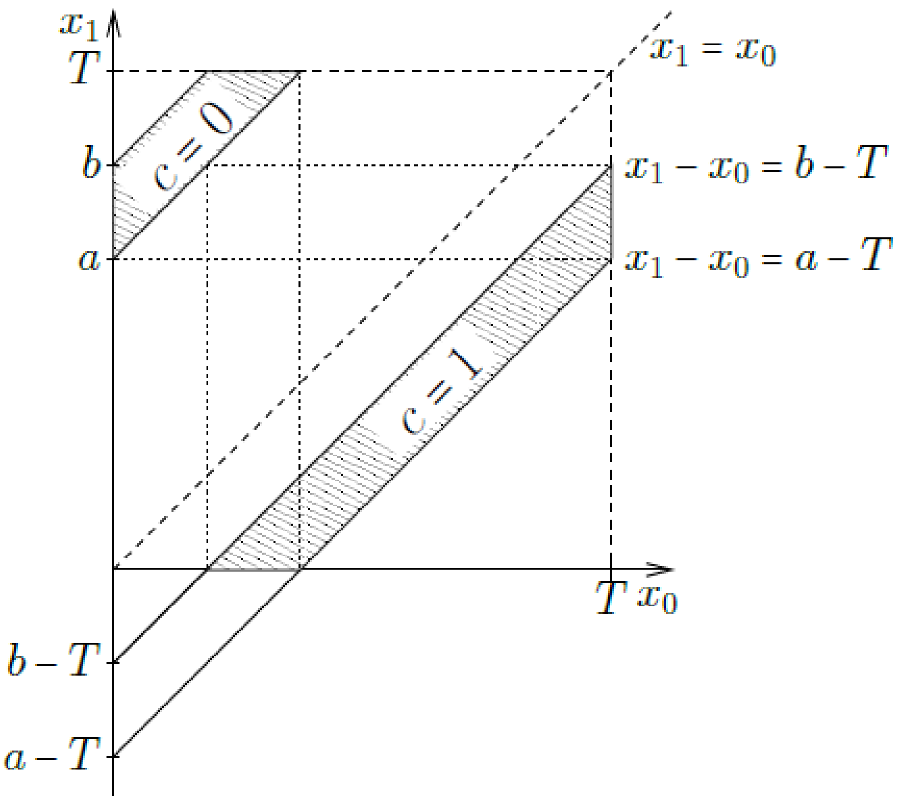


$$\begin{aligned} -z \quad -T \cdot c &\leq -a \\ z \quad +T \cdot c &\leq b+T \end{aligned}$$

$$c \in \{0;1\}$$

Secondary Offsets

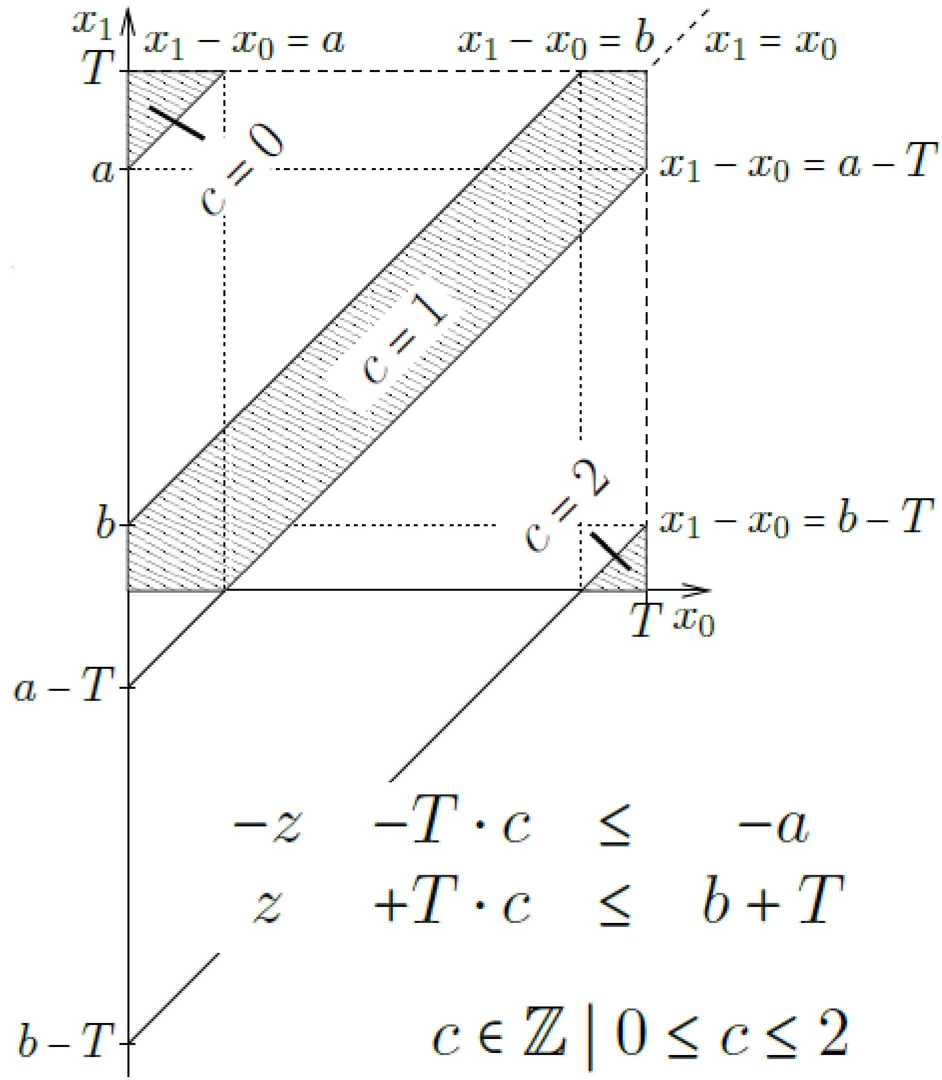
Case $b \geq a$



$$\begin{aligned} -z \quad -T \cdot c &\leq -a \\ z \quad +T \cdot c &\leq b \end{aligned}$$

$$c \in \{0; 1\}$$

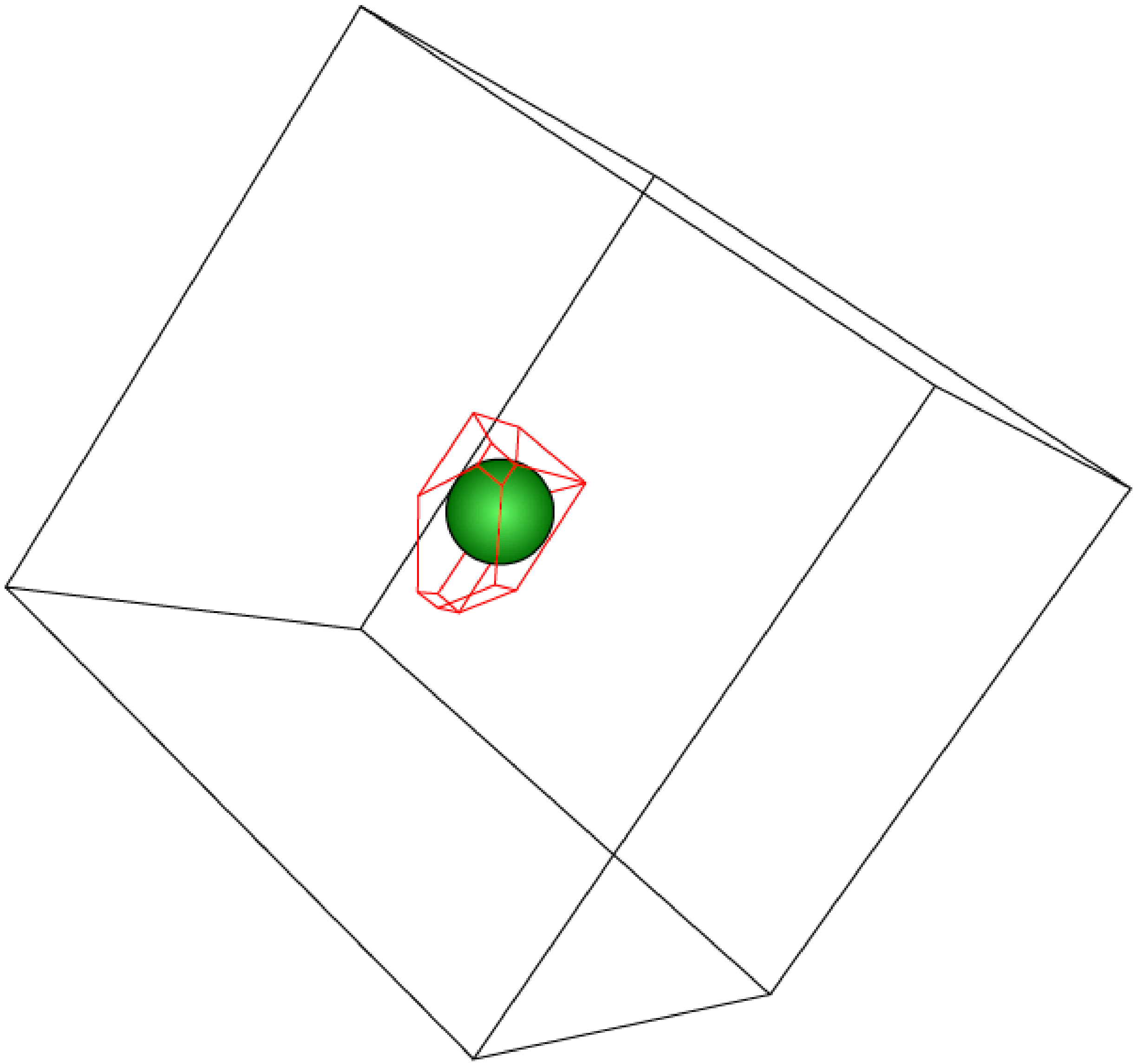
Case $b < a$



$$\begin{aligned} -z \quad -T \cdot c &\leq -a \\ z \quad +T \cdot c &\leq b + T \end{aligned}$$

$$c \in \mathbb{Z} \mid 0 \leq c \leq 2$$

The Shape of the Perfect Solution



Relaxation & Optimization

Case z is primary offset:

Case $b > a$:

$$\begin{array}{rrrcr} -z & & -T\cdot p & \leq & -a \\ z & & -T\cdot p & \leq & b \end{array}$$

Case $b < a$:

$$\begin{array}{rrrcr} -z & -T\cdot c & -T\cdot p & \leq & -a \\ z & +T\cdot c & -T\cdot p & \leq & b+T \end{array}$$

$$c \in \{0,1\}$$

Case z is secondary offset:

Case $b > a$:

$$\begin{array}{rrrcr} -z & -T\cdot c & -T\cdot p & \leq & -a \\ z & +T\cdot c & -T\cdot p & \leq & b \end{array}$$

$$c \in \{0,1\}$$

Case $b < a$:

$$\begin{array}{rrrcr} -z & -T\cdot c & -T\cdot p & \leq & -a \\ z & +T\cdot c & -T\cdot p & \leq & b+T \end{array}$$

$$c \in \{0,1,2\}$$

$$w = 2^{L-(r-l)-2}$$

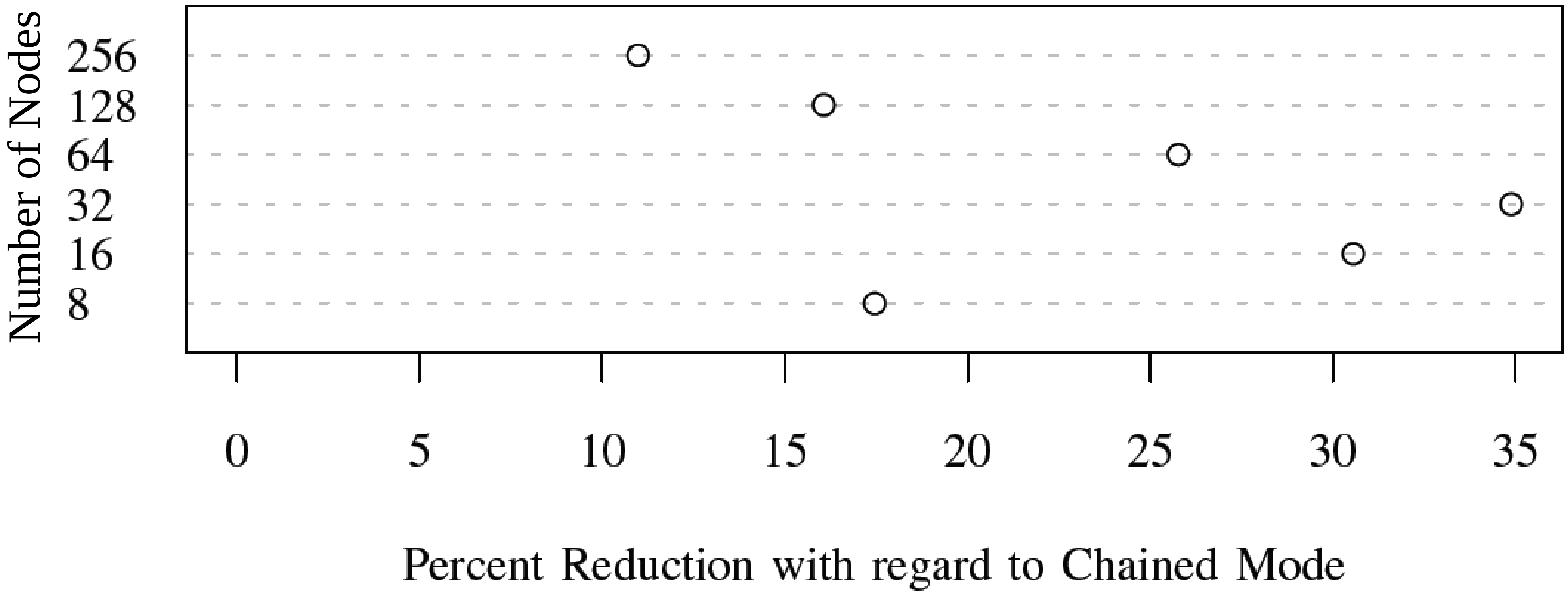
$$obj = T \sum w_e p_e$$

lp_solve

- Open Source Solver
- Michael Berlelaar Eindhoven University of Technology
- Enhanced and updated by various Individuals
- Linear and Mixed Integer Optimizer
  - Uses branch and bound algorithm
  - Presolve algorithms pre-processes the problem to reduce problem size and solve time.
  - Handles binary, integer and semi-continuous variables, and special ordered sets.
  - Customizable node and variable selection strategies.

Results

Reduction of Wait Latency in Crossing Mode for Different Network Sizes, 5 Minutes Time Limit for lp_solve



Summary & Future Work

- Hypercube gossiping facilitates efficient aggregation for a broad range of applications
- Real-time applications require highly current data
- High portion of latency due to wait delay, i.e. data sojourn times at intermediate nodes
- Depends on timing behavior of nodes
- Modeling of Wait-Latency as a MILP-Optimization Problem
- Significant reduction of wait latency
- Performance of the employed solver currently limits perfect solution to 64 nodes

Thank you!